DISCRETE KALMAN FILTER DESIGN FOR MULTIVARIABLE SHIP MOTION CONTROL: EXPERIMENTAL RESULTS WITH TRAINING SHIP

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ABSTRACT

The article presents a concept of Kalman filter which can be used in the multivariable ship motion control system. This system usually measures ship position coordinates and the ship heading, while the velocities are to be estimated using an available mathematical model of the ship. Moreover, the measured variables are burdened with measuring disturbances. The designed Kalman filter was tested in computer simulation tests making use of the mathematical model, and was implemented in the physical model with no goods results.

1. INTRODUCTION

Modern ships are equipped with complicated ship motion control systems, the goals of which depend on tasks realised by an individual ship. The tasks executed by the control system include, among other actions, controlling the ship motion along the course or a given trajectory (path following and trajectory tracking), dynamical positioning and reduction of ship rolls caused by waves. Figure 1 presents basic components of the ship motion control system.

```
Way-points
  v
  | Trajectory generator
  | v
  | Controller
  | v
  | Allocation
  | v
  | Ship
  | v
  | DGPS
  | Gyro-compass

Guidance System

Motion-Control System

Estimated positions and velocities

Navigation System
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Fig. 1. Basic components of modern ship motion control system [5].

The guidance system generates a required smooth reference trajectory, described using given positions, velocities, and accelerations. The trajectory is generated by algorithms which make use of the required and current ship positions, and the mathematical model with complementary information on the executed task and, possibly, the weather.

The control system processes the motion related signals and generates the set values for actuators to reduce the difference between the desired trajectory and the current trajectory. The controller can have a number of operating modes depending on the executed tasks. On some ships and in some operations the required control action can be executed in several ways due to the presence of a number of propellers. Different combinations of actuators can generate the same control action. In those cases the control system has also to solve the control allocation problem, based on the optimisation criteria [5].

The navigation system measures the ship position and the heading angle, collects data from various sensors, such as GPS, log, compass, gyro-compass, radar. The navigation system also checks the quality of the signal,
passes it to the observer system in which the disturbances are filtered out and the ship state variables are calculated. Stochastic nature of the forces generated by the environment requires the use of observers for estimating variables related with the moving ship and for filtering the disturbances in order to use the signals in the ship motion control systems.

Filtering and estimating are extremely important properties in the multivariable control systems. In many cases the ship velocity measurements are not directly available, and the velocity estimates are to be calculated from the position and heading values measured by the observer. Unfortunately, these measurements are burdened with errors generated by environmental disturbances like wind, sea currents and waves, as well as by sensor noise.

In 1960 R.E. Kalman published his famous work in which he described a recurrent solution of a linear, discrete problem of filtering [15]. Since then, following rapid development of computer technology the Kalman filter has become an object of intensive investigations and applications, particularly in the areas of autonomous and controlled navigation. A very clear introduction to the general concept of Kalman filter can be found in Chapter 1 [16], while a more detailed description, complemented by mathematical apparatus is given by [20]. Interesting presentation of the Kalman filter, along with historical overview of its development, is given by [19]. More detailed descriptions concerning the filtration problems and the introduction to stochastic processes can be found in [3, 9, 14].

Initially, the multivariable systems of ship motion control were designed using conventional PID controllers arranged in cascade with low-passing filters and cutting-off filters to remove the wave generated motion components from the control system loop. Since the last century mid-seventies more advanced techniques have appeared, which were based on optimal control and the Kalman filter, see [1]. That solution was further modified and developed by [2, 6, 8, 12, 13, 17, 18]. The Kalman filter can make use of measurements done by different sensors at different accuracy levels, and calculate ship velocity estimates which are not measured in the majority of ship positioning applications.

The main goal of the article is designing and testing the observer for the ship motion velocity estimation.

2. MODEL OF THE PROCESS

A ship sailing on the surface of the water region is considered a rigid body moving in three degrees of freedom. Let the ship position \((x, y)\) and the heading angle relative to the north \(\psi\) in the horizontal plane with respect to the stationary coordinate system (inertial frame) \(\{x, y\}\) be represented by the vector \(\eta = [x, y, \psi]^T\). The other coordinate system (body frame) \(\{x_b, y_b\}\) is fixed to the centre of gravity of the moving ship. Velocities of the moving ship are represented by the vector \(v = [u, v, r]^T\), where \(u\) is the surge velocity of the ship, \(v\) is the sway velocity, and \(r\) is the yaw rate. These variables are shown in Fig. 2.

The position coordinates \((x, y)\) are measured by DGPS (Differential Global Positioning System), while the ship heading \(\psi\) is measured by the gyrocompass. These three state variables are collected in the vector \(\eta = [x, y, \psi]^T\). The three remaining state variables, which are to be estimated, are collected in the vector \(v = [u, v, r]^T\).

The ship motion equations simply express the second Newton’s law of motion in three degrees of freedom. If the descriptions of these equations bases on the stationary coordinate system fixed to the water region map, then the equations of motion take the following form [4]

\[
\begin{align*}
mx' &= X \\
my' &= Y \\
I_{c} \ddot{\psi} &= N
\end{align*}
\]  

where:

- \(X\) – the force acting along \(x_b\) axis,
\( Y \) – the force acting along \( y_b \) axis,
\( N \) – the torque,
\( m \) – the mass of the ship,
\( I_z \) – The moment of inertia around a perpendicular axis directed downward.

\[ \begin{align*}
\text{[Inertial frame]} & \quad x \\
(\text{North}) & \quad x_b \quad \text{(North)} \\
(\text{East}) & \\
\text{[Body frame]} & \quad y \\
& \quad \psi \quad \psi_c \\
& \quad u \quad \beta \\
y_b \quad (\text{Sway}) & \\
& \quad U \\
& \quad y_e \quad (\text{East}) \\
& \quad x_e \\
& \quad \{ \text{Inertial frame} \} \\
& \quad \{ \text{Body frame} \} \\
\end{align*} \]

**Fig. 2.** Ship motion variables.

The above differential equations can be presented in the form of three sets of dynamic equations: for the first degree of freedom

\[
\frac{d}{dt} \begin{bmatrix} u_x \\ x \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_x \\ x \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{X}{m}
\]

\[ x = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ x \end{bmatrix} \] \hspace{1cm} (4a)

for the second degree of freedom

\[
\frac{d}{dt} \begin{bmatrix} u_y \\ y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_y \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{Y}{m}
\]

\[ y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} u_y \\ y \end{bmatrix} \] \hspace{1cm} (5a)

for the third degree of freedom

\[
\frac{d}{dt} \begin{bmatrix} r \\ \psi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r \\ \psi \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{N}{I_z}
\]

\[ \psi = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ \psi \end{bmatrix} \] \hspace{1cm} (6a)

where:

\[ u_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad r = \frac{d\psi}{dt} \quad – \text{velocities in the stationary coordinate system.} \]
The velocity vector \( \mathbf{v} = [u, v, r]^T \) expressed in the moving coordinate system \( \{x_b, y_b\} \), can be calculated from velocities determined in the stationary coordinate system \( \{x_c, y_c\} \) using the relation

\[
\begin{bmatrix}
  u \\
  v \\
  r
\end{bmatrix} =
\begin{bmatrix}
  \cos \psi & \sin \psi & 0 \\
  -\sin \psi & \cos \psi & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u_c \\
  v_c \\
  r
\end{bmatrix}
\]

Equations (4), (5), (6), the dynamic equations for each degree of freedom can be written in the following general way

\[
\dot{x} = Ax + Bu \\
y = Cx
\]

For each degree of freedom matrices \( A, B, C \) are identical and take the form

\[
A = \begin{bmatrix}
0 & 0 \\
1 & 0
\end{bmatrix}, \quad B = \begin{bmatrix} 1 \\
0 \end{bmatrix}, \quad C = [0 \ 1]
\]

### 3. DISCRETE KALMAN FILTER

Rudolph Kalman described a recurrent solution of a linear, discrete filtration problem, which now bears the name of a discrete Kalman filter [15]. We can say, in general, that the Kalman filter attempts to estimate, in an optimal way, the state vector of the controlled process modelled by linear, stochastic difference equation in the form [3]:

\[
x_{k+1} = F_k x_k + G_k w_k
\]

Observations (measurements) of the process are done in discrete times and are described by the following linear relation:

\[
y_k = H_k x_k + v_k
\]

where \( x_k \) is the state vector of the process in time \( t_k \), \( F_k \) is the matrix relating \( x_k \) to \( x_{k+1} \) at the absence of the excitation function, \( w_k \) are accidental disturbances affecting the process, \( G_k \) is the matrix scaling the amplitude of the process disturbances, \( y_k \) is the vector of values measured at time \( t_k \), \( H_k \) is the matrix which represents the connections between the measurements and the state vector at time \( t_k \), \( v_k \) describes the measurement errors. The signals \( v_k \) and \( w_k \) are assumed to have the average values equal to zero, with no correlation between these quantities.

The covariance matrices for vectors \( w_k \) and \( v_k \) are defined as

\[
E\{w_k w_k^T\} = \begin{cases}
  Q_k, & i = k \\
  0, & i \neq k
\end{cases}
\]

\[
E\{v_k v_k^T\} = \begin{cases}
  R_k, & i = k \\
  0, & i \neq k
\end{cases}
\]

\[
E\{w_k v_k^T\} = 0, \quad \text{for all } k \text{ and } i
\]
We assume that the initial values of the process estimates at an arbitrary initial time $t_k$ are known and that these estimates base on the knowledge on the process until the time $t_k$. This estimate will be denoted as $\tilde{x}_k$ where the upper dash means that this is the best estimate before the measurement at time $t_k$. The estimation error is defined as

$$\bar{e}_k = x_k - \tilde{x}_k$$  \hspace{1cm} (15)

and the error covariance matrix referring to this difference is

$$\bar{P}_k = E\left\{ \bar{e}_k \bar{e}_k^T \right\} = E\left\{ (x_k - \tilde{x}_k)(x_k - \tilde{x}_k)^T \right\}$$  \hspace{1cm} (16)

At the sampling times $t_k$, at which the measurement $y_k$ is done, the estimate $\tilde{x}_k$ will be updated according to the following relation [3, 7]

$$\hat{x}_k = \bar{x}_k + L_k(y_k - H_k\bar{x}_k)$$  \hspace{1cm} (17)

where $\hat{x}_k$ is the estimate updated by the recorded measurement, and $L_k$ is the scaling gain. Now the task is to find gains for the vector $L_k$ which will modify the estimate in the optimal way. For this purpose the minimisation of the mean square error is done. Then the error covariance matrix referring to the estimate updated by the recorded measurement is defined:

$$\bar{P}_k = E\left\{ e_k e_k^T \right\} = E\left\{ (x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \right\}$$  \hspace{1cm} (18)

In the time periods between the sampling times the estimates are calculated using the following formula

$$\bar{x}_{k+1} = F_k \hat{x}_k$$  \hspace{1cm} (19)

First, the covariance matrix $\bar{P}_{k+1}$ is calculated based on the formula (16) after correcting it by one sample forward:

$$\bar{P}_{k+1} = E\left\{ (x_{k+1} - \bar{x}_{k+1})(x_{k+1} - \bar{x}_{k+1})^T \right\}$$  \hspace{1cm} (20)

After placing relations (10) and (19) into formula (20) we get:

$$\bar{P}_{k+1} = E\left\{ F_k (x_k - \hat{x}_k) + G_k w_k \right\} (F_k (x_k - \hat{x}_k) + G_k w_k)^T + F_k E\left\{ (x_k - \hat{x}_k)w_k^T \right\} + G_k E\left\{ w_k (x_k - \hat{x}_k)^T \right\} W_k^T$$  \hspace{1cm} (21)

We assume no correlation between the estimation error signals $e_k$ and the disturbances $w_k$, and after placing the covariances defined by formulas (12) and (18) into relation (21) we obtain the wanted error covariance matrix between the sampling times

$$\bar{P}_{k+1} = F_k \bar{P}_k F_k^T + G_k Q_k G_k^T$$  \hspace{1cm} (22)

In a similar way we derive the estimation error covariance matrix $P_k$ for the sampling times. After placing relations (17) and (11) into formula (18) we get

$$P_k = E\left\{ (x_k - \bar{x}_k)(x_k - \bar{x}_k)^T \right\}$$  \hspace{1cm} (23)

Now, doing similar operations as with formula (21) we arrive at the following solution

$$P_k = \bar{P}_k - L_k H_k \bar{P}_k - \bar{P}_k H_k^T L_k^T + L_k (H_k \bar{P}_k H_k^T + R_k) L_k^T$$  \hspace{1cm} (24)
The next quantities to be calculated are the optimal values of the gain matrix \( \mathbf{L}_k \). This task will be executed by finding such values of the vector \( \mathbf{L}_k \) which minimise the trace of the matrix \( \mathbf{P}_k \), which is the sum of mean-square errors of the estimates of all state vector elements. Then the trace of the matrix \( \mathbf{P}_k \) is differentiated with respect to \( \mathbf{L}_k \) and made equal to zero. It is easy to see that the second and third term of equation (24) are linear with respect to \( \mathbf{L}_k \) while the fourth term is quadratic, and that the trace \( \mathbf{L}_k \mathbf{H}_k \mathbf{P}_k \) is equal to the trace of its transposition \( \mathbf{P}_k \mathbf{H}_k^T \mathbf{L}_k^T \).

\[
\frac{d}{d \mathbf{K}_k} \left( \text{trace} \mathbf{P}_k \right) = -2 \left( \mathbf{H}_k \mathbf{P}_k \right) + 2 \mathbf{L}_k \left( \mathbf{H}_k^T \mathbf{P}_k^T + \mathbf{R}_k \right) = 0
\]  

(25)

and after some transformations we get the wanted matrix \( \mathbf{L}_k \), bearing the name of the Kalman gain

\[
\mathbf{L}_k = \mathbf{P}_k \mathbf{H}_k^T \left( \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T + \mathbf{R}_k \right)^{-1}
\]

(26)

Finally, we have to remove the matrix \( \mathbf{L}_k \) from equation (24) by placing the relation from equation (26). Then we get

\[
\mathbf{P}_k = \mathbf{P}_k - \mathbf{P}_k \mathbf{H}_k^T \left( \mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T + \mathbf{R}_k \right) \mathbf{H}_k \mathbf{P}_k
\]

(27)

Figure 3 presents, in the form of a block diagram, a recurrent calculation algorithm based on equations (17), (19), (22) (26) and (27), which is commonly known as the Kalman filter.
4. EXPERIMENTAL RESULTS WITH THE DESIGNED DISCRETE KALMAN FILTER

The model described by formula (8) makes the basis for designing the Kalman filter. For the algorithm of the discrete Kalman filter to be used, the mathematical model (8) is to be digitised to the form given by formulas (10) and (11). Then the matrices $F$ and $H$ take the following forms [7]:

$$
F = e^{AT} = I + AT + \frac{A^2 T^2}{2!} + \frac{A^3 T^3}{3!} + \ldots = \begin{bmatrix} 1 & 0 \\ \end{bmatrix} T 
$$

$$
H = C = \begin{bmatrix} 0 & 1 \\ \end{bmatrix} 
$$

where $T$ is the sampling period.

In order to assess the quality of operation of the designed Kalman filter algorithm, tests were performed both in the form of simulation calculations and experiments done on a physical ship. The experiments were performed on a training ship *Blue Lady* owned by the Foundation for Safety of Navigation and Environment Protection in Ilawa. The ship *Blue Lady* is a physical model, made in the 1:24 scale, of a tanker designed for transporting crude oil. The overall length of *Blue Lady* is $L = 13.75$ m, width $B = 2.38$ m, and mass $m = 22.934 \times 10^3$ [kg]. The system of moving coordinates is fixed to the ship’s centre of gravity [10, 11].

For the purpose of the Kalman filter algorithm, relevant values of the coefficients in the matrices $G$, $Q$ and $R$ were selected. For the first and second degree of freedom the selected values were the following:

$$
G_x = G_y = \begin{bmatrix} 1 & 0 \\ 0 & 0.01 \\ \end{bmatrix}, \quad Q_x = Q_y = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \\ \end{bmatrix}, \quad R_x = R_y = 0.01
$$

while for the third degree of freedom:

$$
G_{\psi} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.01 \\ \end{bmatrix}, \quad Q_{\psi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \end{bmatrix}, \quad R_{\psi} = 0.1
$$

The covariances of the position coordinates measured by GPS $R_x = R_y = 0.01$ and those of the ship course angle $R_{\psi} = 0.1$ were determined based on the experimental tests done on *Blue Lady*. The sampling period was equal to 1 [s].

Initially, the simulation calculations were performed in the Matlab/Simulink environment based on the mathematical model of *Blue Lady*, described in detail by [10, 11]. These tests were performed without measurement disturbances and in these conditions the Kalman filter algorithm described in the article revealed good estimation quality. Due to limited space of the article the results of the simulation tests are not included.

After the simulation tests, the algorithm of the discrete Kalman filter was implemented on the *Blue Lady* model sailing on the lake Silm near Ilawa. Experimental tests were performed to assess the operating quality of the filter and its insensitivity to disturbances.

The results recorded in these tests are shown in figures 4 - 6. As can be noticed, the estimates of the ship position and the heading angle are the same as the measured values. Unfortunately, the time-histories of the estimated parameters look much worse, as they are not smooth, and are burdened with very large errors. All inaccuracies included in the measured values manifest themselves in the estimated velocity values.
Rys. 4. Experimental data: actual position with estimate and estimated velocity in surge.

Rys. 5. Experimental data: actual position with estimate and estimated velocity in sway.

Rys. 6. Experimental data: heading angle with estimate and estimated angular rate in yaw.
5. CONCLUSIONS

The article presents the results of examination of the discrete Kalman filter algorithm in the context of estimation of the state variable vector describing the motion of a ship. The ship velocities are estimated from the measured noised \(x, y\) coordinates of ship position and the ship course \(\psi\). In the measurements of the position coordinates done by GPS stepwise shifts are observed along with additive errors, which manifest themselves as the oscillations in the estimated velocities, see Figs. 4 and 5. The recorded experimental results reveal that the quality of the ship velocity estimation is very low, and the measurement disturbances can be observed in the time histories. The possible source of this poor quality of the ship velocity estimation is that the algorithm of the discrete Kalman filter does not make use of the mathematical model of ship dynamics but only bases on the measurements alone when calculating the velocity estimates.

REFERENCES